Eur. Phys. J. C 52, 905–917 (2007)
DOI 10.1140/epjc/s10052-007-0420-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Searching for new physics in future neutrino factory experiments

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Received: 26 July 2007 /
Published online: 16 October 2007 –

Abstract. An extension of the new standard model, by introducing a mixing of the low mass "active" neutrinos with heavy ones, or by any model with lepton flavor violation, is considered. This leads to non-orthogonal neutrino production and detection states and to modifications of neutrino oscillations in both vacuum and matter. The possibility of the discovery of such effects in current and future neutrino oscillation experiments is discussed. First order approximation formulas for the flavor transition probabilities in constant density matter, for all experimentally available channels, are given. Numerical calculations of flavor transition probabilities for two sets of new physics parameters describing a single "effective" heavy neutrino state, both satisfying present experimental constraints, have been performed. Two energy ranges and several baselines, assuming both the current ($\pm 2\sigma$) and the expected future errors ($\pm 3\%$) of the neutrino oscillation parameters are considered, keeping their present central values. It appears that the biggest potential of the discovery of the possible presence of any new physics is pronounced in oscillation channels in which ν_e and $\nu_{\bar{e}}$ are not involved at all, especially for two baselines, $L = 3000$ km and $L = 7500$ km, which for other reasons are also called "magic" for future Neutrino Factory experiments.

PACS. 13.15.+g; 14.60.Pq; 14.60.St

1 Introduction

For several years, neutrinos have been considered to be massive particles [1–13, 15, 16][1], and therefore the orthodox standard model (SM) with massless neutrinos must be extended. There exist two possibilities. Firstly, the extension of the SM can appear only at a very high energy scale, the GUT/Planck scale, and the non-zero neutrino masses are just the visible indication of such "high energy" physics in our "low energy" world. Such a scenario is usually called the new standard model (νSM). Secondly, some new physics (NP) may already be present at the TeV scale, that means, at energies close to our present-day experimental facilities. The second of these possibilities is more appealing from both the experimental and the theoretical points of view. Such a "low energy" NP can participate in neutrino flavor transitions, so it could possibly be measured in future neutrino oscillation experiments. Then everything is dependent on the precision of the planned experiments. The bounds on the NP parameters, which arise from today's experiments, are too restrictive to give any good chance to see any effects in the present neutrino flavor transition data, taking into account the fact that the present precision in the determination of the neutrino oscillation parameters (of about 10% [17–20]) effectively screens off any possible presence of NP. However, the combined expected results from future neutrino facilities, like Beta Beam, Super Beam, and Neutrino Factory, should bring the neutrino oscillation parameter errors down to about 1%–3% [21, 22] and therefore give a chance for the discovery of effects that possibly could not be explained by the "present physics".

The potential for the NP discovery is considered in this paper. There are many ways in which NP can modify neutrino oscillations. The non-standard effects can directly change oscillation probabilities (the so called "damping signatures" [23, 24]), or they can modify the oscillation amplitudes (by non-standard Hamiltonian effects [25–82]), where both oscillations in vacuum and in matter can be affected. These possibilities have extensively been examined in the existing literature. Thus, there are models with sterile [25–28] or/and heavy [29] neutrinos, general models with lepton flavor violation (LFV) [30, 31], non-standard interactions [32–40], flavor changing neutral currents [41–43], general fermion interactions [44] and mass varying neutrinos [45–49]. Next, there are models with non-unitary leptonic mixing [50, 51], violation of the Lorentz symmetry [52, 53], violation of the principle of general relativity [54–57] and violation of the CPT symmetry [58–60]. Finally, there are models that modify neutrino oscillations, i.e. models with neutrino wave packet decoherence [61–65], neutrinos' decays [66–73] and neutrino quantum decoherence [74–82].

In this paper, we discuss one class of NP only, which can be obtained by mixing of the low mass "active" neutrinos

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[1] Reviews of solar neutrinos include J. Bahcall's URL [14].

with heavy ones [29], or by any model with LFV [30, 31], in both of which neutrinos interact with matter particles by the left-handed charge and neutral currents only. In such models, the effective mixing matrix is non-unitary, resulting in non-orthogonal neutrino production and detection states. This non-orthogonality by itself modifies neutrino oscillations in vacuum. Apart from this, the neutrino interactions with matter particles are non-standard, so the oscillation effects in matter are further modified as well. Both these effects are here taken into account. Additionally, in our numerical calculations, we assume both $\pm 3\%$ and $\pm 2\sigma$ errors of today's νSM neutrino oscillation parameters, keeping their present central values (see Sect. 3). The flavor transition probabilities were calculated for two energy ranges and several baselines [83, 84]. Two of these baselines, $L = 3000$ km and $L = 7500$ km, are called "magic" for future Neutrino Factory experiments, as they are especially useful for CP violation discovery ($L = 3000$ km) [85] or optimal for resolving the oscillation parameters degeneracy problem ($L = 7500$ km). We have performed our numerical calculations of flavor oscillation probabilities for all available channels for two sets of NP parameters that describe a single "effective" heavy neutrino state, both satisfying present experimental constraints. One of the easiest channels, from the experimental point of view [86, 87], is the $\nu_\mu \to \nu_e$ one, but it will be difficult to observe any NP there (as it will also be in all another channels in which ν_e or $\nu_{\bar{e}}$ are involved). In two other channels, $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_\mu$ (and in the corresponding antineutrino channels), the effects of NP are seen with the largest intensity, especially for the "magic" baselines mentioned above [88]. In the next section we investigate how the NP modifies νSM oscillation transition probabilities. We discuss all leading terms, which give the new non-orthogonal production and detection states and which modify the neutrino coherent scattering on matter particles. We also discuss the reason why all channels with ν_μ or ν_τ as the initial and final neutrinos (and the corresponding antineutrino channels), i.e. all channels in which ν_e and $\nu_{\bar{e}}$ are not involved at all, are the most desired ones. Then, in Sect. 3, we present the results of our numerical simulations. And finally, in Sect. 4, we give our conclusions. In the appendix we collect all formulas for the flavor transition probabilities in constant density matter for all experimentally available channels.

2 Searching for new physics in neutrino oscillation experiments

If one takes into account that only relativistic neutrinos are detected (and that only left-handed vector interactions are considered), the detection rate $N_{\beta\alpha}$ of the ν_β neutrinos, coming from the produced ν_α neutrinos, factorizes into three parts, the production flux N_α, the transition probability $P_{\alpha\to\beta}$, and the detection cross section σ_β:

$$N_{\beta\alpha} = N_\alpha P_{\alpha\to\beta}\sigma_\beta \,. \tag{1}$$

Any physics beyond the νSM will modify all of these three parts (or even the above factorization will be made impossible). In this paper, we only discuss the modifications of the probability of neutrino flavor transition from a production state $|\nu_{\rm P}\rangle$ to the detection state $|\nu_{\rm D}\rangle$, leaving the modifications of the production and detection neutrino cross sections for detailed considerations in the future.

In the frame of the νSM, the production or detection neutrino states are equal to the appropriate orthonormal neutrino flavor states $|\nu_\alpha\rangle$. The NP modifies this dependence and therefore the $|\nu_{\rm P,D}\rangle$ states are only approximately equal to the $|\nu_\alpha\rangle$ states. Let us assume that the neutrinos are produced in the following process:

$$\ell + X \to \nu + Y \,, \tag{2}$$

where ℓ is a charged lepton ($\ell = e, \mu, \tau$), and X and Y are hadrons. Then the normalized neutrino production state $|\nu_{\rm P}\rangle$ can be defined as

$$|\nu_{\rm P}\rangle = \frac{\sum_{i=1}^{n} A(\ell + X \to \nu_i + Y)|\nu_i\rangle}{\sqrt{\sum_{i=1}^{n} |A(\ell + X \to \nu_i + Y)|^2}} \,, \tag{3}$$

where $A(\ell + X \to \nu_i + Y)$ is the amplitude for the process (2), in which the neutrino eigenmass state $|\nu_i\rangle$ is produced. The sum in (3) goes over all neutrinos with masses m_i that are kinematically allowed. If particle spins were taken into account, instead of the pure states of (3), we would have to use mixed states described by an appropriate density matrix.

Let us consider a NP model, in which besides three light νSM neutrinos there are also heavier ones, which couple to the light charged leptons in a non-negligible way [29]. To be more precise, we assume the charge current Lagrangian in the following form ($n > 3$):

$$\mathcal{L}_{\rm CC} = \frac{e}{2\sqrt{2}\sin\theta_{\rm W}} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{n} \psi_\alpha \gamma^\mu (1-\gamma_5)(\mathbf{U}_\nu)_{\alpha i}\nu_i W_\mu^- + \text{h.c.} \,, \tag{4}$$

and similarly the neutral current Lagrangian in the following form:

$$\mathcal{L}_{\rm NC} = \frac{e}{2\sin(2\theta_{\rm W})} \sum_{i,j} \overline{\nu_i}\gamma^\mu (1-\gamma_5)\Omega_{ij}\nu_j Z_\mu \,, \tag{5}$$

where

$$\Omega_{ij} = \sum_{\alpha=e,\mu,\tau} (\mathbf{U}_\nu)^*_{\alpha i}(\mathbf{U}_\nu)_{\alpha j} \,. \tag{6}$$

The $n \times n$ matrix $\mathbf{U}_\nu$ defines the mixing between the flavor and mass states. So, e.g. assuming three light and three heavy neutrinos, we have

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} \mathcal{U}_{\alpha i} & V_{\alpha I} \\ V'_{ai} & \mathcal{U}'_{aI} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3 \end{pmatrix} , \tag{7}$$

where all submatrices ($\mathcal{U}$, V, V' and $\mathcal{U}'$) have dimensions 3×3. The three additional neutrino states, N_e,N_μ and N_τ, do not couple to charged leptons. For Majorana neutrinos the submatrices $\mathcal{U}$ and V, which explicitly enter the interaction Lagrangian (4), depend on $3n-6$ moduli and $3n-6$ CP violating phases. For Dirac neutrinos $n-1$ phases can be eliminated, giving altogether $2n-5$ phases, which, in principle, can enter the NP neutrino flavor transition probabilities. If we assume that in the process given by (2) the energy conservation does not allow one to produce heavy neutrinos N_i, then, according to (3), the neutrino production state ($|\nu_{\rm P}\rangle$) is given by

$$|\nu_{\rm P}\rangle = \frac{1}{\sqrt{\sum_{i=1}^{3}|\mathcal{U}_{\ell i}|^2}}\sum_{i=1}^{3}\mathcal{U}_{\ell i}^*|\nu_i\rangle\,. \tag{8}$$

Such states are normalized but not orthogonal. As the mixing of heavy neutrinos is small ($|V_{\alpha I}|^2 \ll 1$), the matrix $\mathcal{U}$ is almost unitary. If we assume that a matrix U describes a unitary transition and is parameterized by the standard three mixing angles θ_{12}, θ_{13} and θ_{23} and one standard Dirac CP breaking phase δ_{13}, then the orthonormal neutrino flavor state $|\nu_\alpha\rangle$ is the following combination of neutrino mass states $|\nu_i\rangle$:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3}U_{\alpha i}^*|\nu_i\rangle\,. \tag{9}$$

The $\mathcal{U}$ matrix can be parameterized by a matrix Λ close to the unit matrix $\mathbf{1}$:

$$\mathcal{U} = \Lambda U\,, \quad \text{with } \Lambda = \mathbf{1}-\delta\Lambda\,, \tag{10}$$

and therefore the production state $|\nu_{\rm P}\rangle$ in (3) is close to the eigenflavor state $|\nu_\alpha\rangle$ and can also be decomposed in the orthonormal flavor basis:

$$|\nu_{\rm P}\rangle \equiv |\widetilde{\nu_\ell}\rangle = \sum_{\alpha=e,\mu,\tau} d_{\ell\alpha}^*|\nu_\alpha\rangle\,, \tag{11}$$

where the $d_{\ell\alpha}$ parameters are equal to

$$d_{\ell\alpha} = \frac{\Lambda_{\ell\alpha}}{\sqrt{\sum_{i=1}^{3}|\mathcal{U}_{\ell i}|^2}}\,. \tag{12}$$

In the general case, the values of the parameters $(\mathbf{1}-\Lambda)_{\ell\alpha} = (\delta\Lambda)_{\ell\alpha} \equiv \delta\lambda_{\ell\alpha}$ depend on the production (detection) process [30, 31, 89–92], and are bounded by the existing charged lepton data. The same parameterization as in (11) was considered in [30, 31], where the general lepton flavor violation NP model is discussed. In each row of the $d_{\ell\alpha}$ matrix, practically only one element has a non-negligible value, namely

$$|d_{\ell\ell}| \le 1\,, \text{ and } |d_{\ell\alpha}| \approx 0\,, \quad \text{for } \alpha \neq \ell\,. \tag{13}$$

In general, however, the 3×3 matrix $\delta\Lambda$ can have all elements non-vanishing. Therefore, nine moduli and nine phases can generally parameterize any kind of NP. Not all phases play a role in the transition probabilities. Five Majorana type phases do not enter any transition probability formula, hence only four phases remain.

In the model considered here, the elements of the $\delta\Lambda$ matrix are connected with the heavy neutrino mixing matrix V. From the unitary condition for the full $\mathbf{U}_\nu$ matrix we get the following relation between the Λ and V matrices:

$$\Lambda\Lambda^\dagger = \mathbf{1} - VV^\dagger\,, \tag{14}$$

so, neglecting the $\delta\Lambda\delta\Lambda^\dagger$ term, we have

$$\delta\Lambda + \delta\Lambda^\dagger = VV^\dagger\,, \tag{15}$$

or, explicitly,

$$\delta\lambda_{\alpha\beta} + \delta\lambda_{\beta\alpha}^* = c_{\alpha\beta}\,, \quad c_{\alpha\beta} = (VV^\dagger)_{\alpha\beta}\,. \tag{16}$$

Note that these are not explicit formulas for individual $\delta\lambda_{\alpha\beta}$ elements. However, terms in the form of the left-hand sides of (15) and (16) may often entirely describe the NP effects for modified matter oscillations (see (18) and (39) below). If we need the knowledge of individual $\delta\lambda_{\alpha\beta}$ elements, for example in order to calculate the new production and detection neutrino state modifications, then they need to be calculated from (10) (see (22) below and the appendix).

Up to now we have only considered the modifications of the initial and final neutrino states in the probability formula. Such modifications will change the neutrino propagation even in vacuum. Yet, if neutrinos pass matter additional effects arise. The coherent neutrino scattering on matter particles is modified by NP and because of this (i) neutrinos acquire different effective masses and (ii) their coherent scattering amplitude is modified. These effects can be parameterized by a NP effective Hamiltonian $\mathbf{H}^{\rm NP}$ [93–95]. The matrix representation of the $\mathbf{H}^{\rm NP}$ operator depends on the basis of states. Generally, in the basis of states of produced and detected neutrinos, the $\mathbf{H}^{\rm NP}$ operator is not represented by a hermitian matrix. However, it is represented by a hermitian matrix in the eigenmass basis ($|\nu_i\rangle$) and in any basis that is unitary transformed, e.g. in the basis of eigenflavor orthonormal neutrino states given by (9). Therefore, in the orthonormal flavor basis ($|\nu_\alpha\rangle$), we can write

$$\mathbf{H}^{\rm NP} = \frac{A_e}{2E_\nu}\begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu}{\rm e}^{{\rm i}\chi_{e\mu}} & \varepsilon_{e\tau}{\rm e}^{{\rm i}\chi_{e\tau}} \\ \varepsilon_{e\mu}{\rm e}^{-{\rm i}\chi_{e\mu}} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau}{\rm e}^{{\rm i}\chi_{\mu\tau}} \\ \varepsilon_{e\tau}{\rm e}^{-{\rm i}\chi_{e\tau}} & \varepsilon_{\mu\tau}{\rm e}^{-{\rm i}\chi_{\mu\tau}} & \varepsilon_{\tau\tau} \end{pmatrix}\,, \tag{17}$$

where $A_e = 2\sqrt{2}G_{\rm F}N_eE_\nu$ is the usual effective amplitude of the neutrino, which depends on the electron matter density N_e, and $\varepsilon_{\alpha\beta}$ and $\chi_{\alpha\beta}$ are NP parameters, moduli and phases, which describe the effective NP neutrino interaction with the matter particles e, p and n. In the general case these parameters depend in a complicated way on the NP and matter properties. For uncharged, unpolarized and isotropic matter, the parameters ε are connected with the c parameters in a simple way (see (19) below). For example, in the frame of the model that we consider ((4)

and (5) [29]), the effective NP Hamiltonian given by (17) can be determined and, neglecting second order terms in $\delta\Lambda$, is equal to

$$\mathbf{H}^{\mathrm{NP}} = \frac{1}{2E_\nu}\left(- A_e[E(1)\delta\Lambda + \delta\Lambda^\dagger E(1)] + A_n[\delta\Lambda + \delta\Lambda^\dagger]\right), \tag{18}$$

where $A_n = \sqrt{2}G_\mathrm{F} N_n E_\nu$ depends on the neutron matter density N_n (in earth matter $A_n/A_e \approx 1/2$) and $E(1)_{\alpha\beta} = \delta_{\alpha e}\delta_{\beta e}$. Now, by comparing (17) and (18), we can find the following connection between their parameters ($\beta \geq \alpha$ and $\chi_{\alpha\alpha} \equiv 0$; it may often be the case that $\delta\lambda_{e\mu} = \delta\lambda_{e\tau} = 0$, and then $c_{\alpha\beta}$ entirely describe the NP effects for modified matter oscillations – compare (39) below):

$$\varepsilon_{\alpha\beta}\mathrm{e}^{\mathrm{i}\chi_{\alpha\beta}} = \left(\frac{A_n}{A_e} - \delta_{\alpha e}\delta_{\beta e}\right)c_{\alpha\beta} - \delta_{\alpha e}(1-\delta_{\beta e})\delta\lambda_{e\beta}\,. \tag{19}$$

For high energy neutrino beams with $E_\nu \approx O(\mathrm{GeV})$, the following two small parameters, which describe νSM neutrino oscillations, are important (for lower energies, $E_\nu \approx O(\mathrm{MeV})$, a third small factor, $A_e/\delta m^2_{31}$, would enter into the game, too):

$$\alpha = \frac{\delta m^2_{21}}{\delta m^2_{31}} \approx \pm 0.03 \quad \text{and} \quad \sin^2(2\theta_{13}) \leq 0.05\,. \tag{20}$$

Also all $c_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ parameters, which describe NP, are small. Therefore, we can expand the neutrino oscillation probabilities in these small quantities, keeping only the leading first order terms. In this approximation, the full transition probability, for any flavor and the baseline L, can be decomposed into two terms, the νSM probability, and the correction to it given by NP:

$$P_{P(\alpha)\to D(\beta)}(L) = P^{\mathrm{SM}}_{\alpha\to\beta}(L) + \delta P^{\mathrm{NP}}_{\alpha\to\beta}(L)\,. \tag{21}$$

The NP correction probability we decompose again into two terms, the c term, which is responsible for the initial and final neutrino state modifications (see the appendix), and the ε term, which takes into account the NP influence on the coherent neutrino scattering in matter:

$$\delta P^{\mathrm{NP}}_{\alpha\to\beta}(L) = \delta P^{c}_{\alpha\to\beta}(L) + \delta P^{\varepsilon}_{\alpha\to\beta}(L)\,. \tag{22}$$

The ε term also consists of two terms. The first one, which is responsible for an effective neutrino mass change, and the second one, which describes the additional NP impact on coherent neutrino scattering with matter particles, are

$$\delta P^{\varepsilon}_{\alpha\to\beta}(L) = \delta P^{\mathrm{mass}}_{\alpha\to\beta}(L) + \delta P^{\mathrm{int}}_{\alpha\to\beta}(L)\,. \tag{23}$$

Generally, the production and detection states are not orthogonal:

$$\langle \nu_{P(\alpha)} | \nu_{D(\beta)} \rangle \neq \delta_{\alpha\beta}\,, \tag{24}$$

and as a consequence the probability of the neutrino oscillation is not conserved:

$$\sum_{\text{all } \beta} P_{P(\alpha)\to P(\beta)} \neq 1\,. \tag{25}$$

However, the neutrino oscillation probability of the νSM is normalized to 1:

$$\sum_{\text{all } \beta} P^{\mathrm{SM}}_{\alpha\to\beta} = 1\,, \tag{26}$$

whereas the other terms satisfy

$$\sum_{\text{all } \beta} \delta P^{c}_{\alpha\to\beta} \neq 0\,,$$
$$\sum_{\text{all } \beta} \delta P^{\mathrm{mass}}_{\alpha\to\beta} = 0, \quad \sum_{\text{all } \beta} \delta P^{\mathrm{int}}_{\alpha\to\beta} = 0\,. \tag{27}$$

In the next section neutrino propagation in the earth matter will be discussed. In our numerical calculations of neutrino flavor transitions we use the realistic PREM I [96, 97] earth density profile model. However, explicit analytical formulas for the flavor transition probabilities can only be given for the case of constant density matter. Both NP corrections, which are important for neutrino transitions in matter, $\delta P^{\mathrm{int}}_{\beta\to\gamma}(L)$ and $\delta P^{\mathrm{mass}}_{\beta\to\gamma}(L)$, are small; therefore, their linear decomposition in terms of α and $\sin(2\theta_{13})$ is a very good approximation (in all formulas, we assume $\delta m^2_{21} = \delta m^2_{\mathrm{sol}}$ and $\delta m^2_{31} = \pm\delta m^2_{\mathrm{atm}} + \delta m^2_{\mathrm{sol}}/2$, where the upper/lower sign refers to the normal/inverted mass hierarchy [98, 99]):

$$\delta P^{\mathrm{int}}_{\beta\to\gamma} = B^0_{\beta\gamma} + \alpha B^\alpha_{\beta\gamma} + \sin(2\theta_{13}) B^s_{\beta\gamma}\,, \tag{28}$$

and similarly

$$\delta P^{\mathrm{mass}}_{\beta\to\gamma} = C^0_{\beta\gamma} + \alpha C^\alpha_{\beta\gamma} + \sin(2\theta_{13}) C^s_{\beta\gamma}\,. \tag{29}$$

The largest terms that are not suppressed, neither by α nor by $\sin(2\theta_{13})$, namely the terms $B^0_{\beta\gamma}$ and $C^0_{\beta\gamma}$, do not appear in any ν_e nor in $\nu_{\bar{e}}$ related channels. Such terms are only present in the $\nu_\mu \to \nu_\tau$, $\nu_\mu \to \nu_\mu$ and $\nu_\tau \to \nu_\tau$ oscillation channels (and in the corresponding antineutrino oscillation channels). For all such channels they are, up to the sign, the same and have the following form:

$$\begin{aligned} B^0 = \widehat{A}_e \sin(4\theta_{23})\{&\sin(2\theta_{23})(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) \\ &+ 2\cos(2\theta_{23})\cos(\chi_{\mu\tau})\varepsilon_{\mu\tau}\}\sin^2(\Delta)\,, \end{aligned} \tag{30}$$

and

$$\begin{aligned} C^0 = \widehat{A}_e \Delta \sin^2(2\theta_{23})\{&-\cos(2\theta_{23})(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) \\ &+ 2\sin(2\theta_{23})\cos(\chi_{\mu\tau})\varepsilon_{\mu\tau}\}\sin(2\Delta)\,, \end{aligned} \tag{31}$$

where

$$\widehat{A}_e \equiv \frac{A_e}{\delta m^2_{31}} \quad \text{and} \quad \Delta = \frac{\delta m^2_{31} L}{2E_\nu}\,. \tag{32}$$

For the $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_\mu$ transitions we obtain

$$B^0_{\mu\tau} = -B^0_{\mu\mu} = B^0 \quad \text{and} \quad C^0_{\mu\tau} = -C^0_{\mu\mu} = C^0\,. \tag{33}$$

Unfortunately, these channels, wherein we can expect the largest NP effects, are not easily reached experimentally.

To see the NP effects in channels where they are suppressed, in the next section, we discuss one of the easiest experimental channels, the $\nu_\mu \to \nu_e$ one. All non-leading terms in (28) and (29), together with all terms for antineutrino and time reversal channels, are given in the appendix.

In the model that we discuss, the c parameters are constrained from the existing experimental data [29, 100–104]:

$$\begin{aligned} &c_{ee} \le 0.0054\,, \quad c_{\mu\mu} \le 0.0096\,, \quad c_{\tau\tau} \le 0.016\,, \\ &|c_{e\mu}| = |c_{\mu e}| \le 0.0001\,, \quad |c_{e\tau}| = |c_{\tau e}| \le 0.009\,, \\ &|c_{\mu\tau}| = |c_{\tau\mu}| \le 0.012\,. \end{aligned} \tag{34}$$

There are no constraints on the phases. The ε parameters for the neutrino propagation in matter (17) are determined from the relations given by (19).

These are also the confinements that we use in the next section, where we present some results of our numerical calculations of the differences of the transition probabilities between CP conjugate channels:

$$\Delta P^{CP}_{\alpha\to\beta}(L) = P_{\alpha\to\beta}(L) - P_{\overline{\alpha}\to\overline{\beta}}(L)\,. \tag{35}$$

3 New physics in future neutrino oscillation experiments

In order to check the effects of the NP described above, the probability differences $\Delta P^{CP}_{\alpha\to\beta}$ (35) for two energy ranges and several baselines have been calculated. Both the energy ranges and the baselines have been chosen with the prospect of existing, planned, and feasible experiments. In view of the Beta Beam and Super Beam experiments, the first energy range is $E_\nu = 0.1$–5 GeV and the considered baselines are $L = 130$, 295 and 810 km [105–107]. In view of the Neutrino Factory experiments, the second energy range is $E_\nu = 1$–50 GeV and the considered baselines are $L = 732$, 3000 and 7500 km [108, 109]. In our numerical calculations of the neutrino flavor transitions, we use the realistic PREM I [96, 97] earth density profile model, which assesses the actual matter density ρ and the actual electron fraction Y_e, along the neutrino flight path in the earth's interior. Then we have

$$\begin{aligned} A_e[\mathrm{eV}^2] &= 7.63\times 10^{-5}\left[\frac{\rho}{\mathrm{g/cm}^3}\right]\left[\frac{Y_e}{0.5}\right]\left[\frac{E_\nu}{\mathrm{GeV}}\right], \\ A_n[\mathrm{eV}^2] &= 7.63\times 10^{-5}\left[\frac{\rho}{\mathrm{g/cm}^3}\right][1-Y_e]\left[\frac{E_\nu}{\mathrm{GeV}}\right]. \end{aligned} \tag{36}$$

Note here that for $L \lesssim 874$ km neutrinos pass only the first shell of the earth's crust, with a constant density $\rho = 2.6\,\mathrm{g/cm}^3$ and $Y_e = 0.494$, thus $A_e[\mathrm{eV}^2] = 1.96\times 10^{-4}[E_\nu/\mathrm{GeV}]$ and $A_n[\mathrm{eV}^2] = 1.0\times 10^{-4}[E_\nu/\mathrm{GeV}]$.

The νSM oscillation parameters, together with their $\pm 2\sigma$ errors (95% C.L.; correlations among the parameters are currently considered small), are taken from the current global best fit values [98, 99]:

$$\begin{aligned} \sin^2(\theta_{13}) &= 0.9^{+2.3}_{-0.9}\times 10^{-2}\,, \\ \delta m^2_{\mathrm{sol}} &= 7.92(1\pm 0.09)\times 10^{-5}[\mathrm{eV}^2]\,, \\ \sin^2(\theta_{12}) &= 0.314\left(1^{+0.18}_{-0.15}\right), \\ \delta m^2_{\mathrm{atm}} &= 2.4\left(1^{+0.21}_{-0.26}\right)\times 10^{-3}[\mathrm{eV}^2]\,, \\ \sin^2(\theta_{23}) &= 0.44\left(1^{+0.41}_{-0.22}\right). \end{aligned} \tag{37}$$

In order to implement the effects of heavy neutrinos, as discussed in the previous section, the matrix V must be introduced. As the number of heavy non-decoupling neutrinos is unknown, we parameterize it in a simplified way, using a single "effective" heavy neutrino state. In this way, the number of independent quantities parameterizing the matrix $\mathcal{U}_{\ell i}$ in (8) are six moduli and three CP phases: three standard mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ plus one standard Dirac phase (δ_{13}) plus three new small NP mixing angles $(\theta_{14}, \theta_{24}, \theta_{34})$ plus two new NP Dirac phases $(\delta_{24}, \delta_{34})$. Then

$$V = \begin{pmatrix} \sin(\theta_{14}) \\ \cos(\theta_{14})\sin(\theta_{24})\mathrm{e}^{-\mathrm{i}\delta_{24}} \\ \cos(\theta_{14})\cos(\theta_{24})\sin(\theta_{34})\mathrm{e}^{-\mathrm{i}\delta_{34}} \end{pmatrix}, \tag{38}$$

and

$$\delta\Lambda = \begin{pmatrix} 1-\cos(\theta_{14}) & 0 & 0 \\ \sin(\theta_{14})\sin(\theta_{24})\mathrm{e}^{-\mathrm{i}\delta_{24}} & 1-\cos(\theta_{24}) & 0 \\ \sin(\theta_{14})\cos(\theta_{24}) \times\sin(\theta_{34})\mathrm{e}^{-\mathrm{i}\delta_{34}} & \sin(\theta_{24})\sin(\theta_{34}) \times\mathrm{e}^{-\mathrm{i}(\delta_{34}-\delta_{24})} & 1-\cos(\theta_{34}) \end{pmatrix}. \tag{39}$$

Two sets of V parameters, both satisfying present experimental constraints given by (34), are discussed below:

$$(\mathrm{A}):\ \begin{pmatrix} 0.001 \\ 0.1\mathrm{e}^{-\mathrm{i}\delta_{24}} \\ 0.1\mathrm{e}^{-\mathrm{i}\delta_{34}} \end{pmatrix}, \quad (\mathrm{B}):\ \begin{pmatrix} 0.01 \\ 0.01\mathrm{e}^{-\mathrm{i}\delta_{24}} \\ 0.1\mathrm{e}^{-\mathrm{i}\delta_{34}} \end{pmatrix}. \tag{40}$$

All calculations have been performed assuming the direct mass scheme only. As nothing is known about the values of the CP phases, we allow them to vary freely.

For both V sets, we notice that the biggest potential for the discovery of the possible presence of any NP is pronounced in oscillation channels in which ν_e and $\nu_{\bar{e}}$ are not involved at all; that is, in $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_\mu$ (including the corresponding antineutrino channels). The effects are especially visible for two baselines, $L = 3000$ km and $L = 7500$ km, which, for other reasons, are also considered "magic" for future Neutrino Factory experiments. Moreover, comparing numerical results for these two sets of V parameters, we can clearly see that, as in (40.B) the magnitude of the middle row is ten times smaller than the corresponding magnitude in (40.A), the NP effects for the second V set (40.B) are smaller by a similar factor (compare (41) below), too.

In order to find how the uncertainty of the estimation of the νSM oscillation parameters can mimic any possible

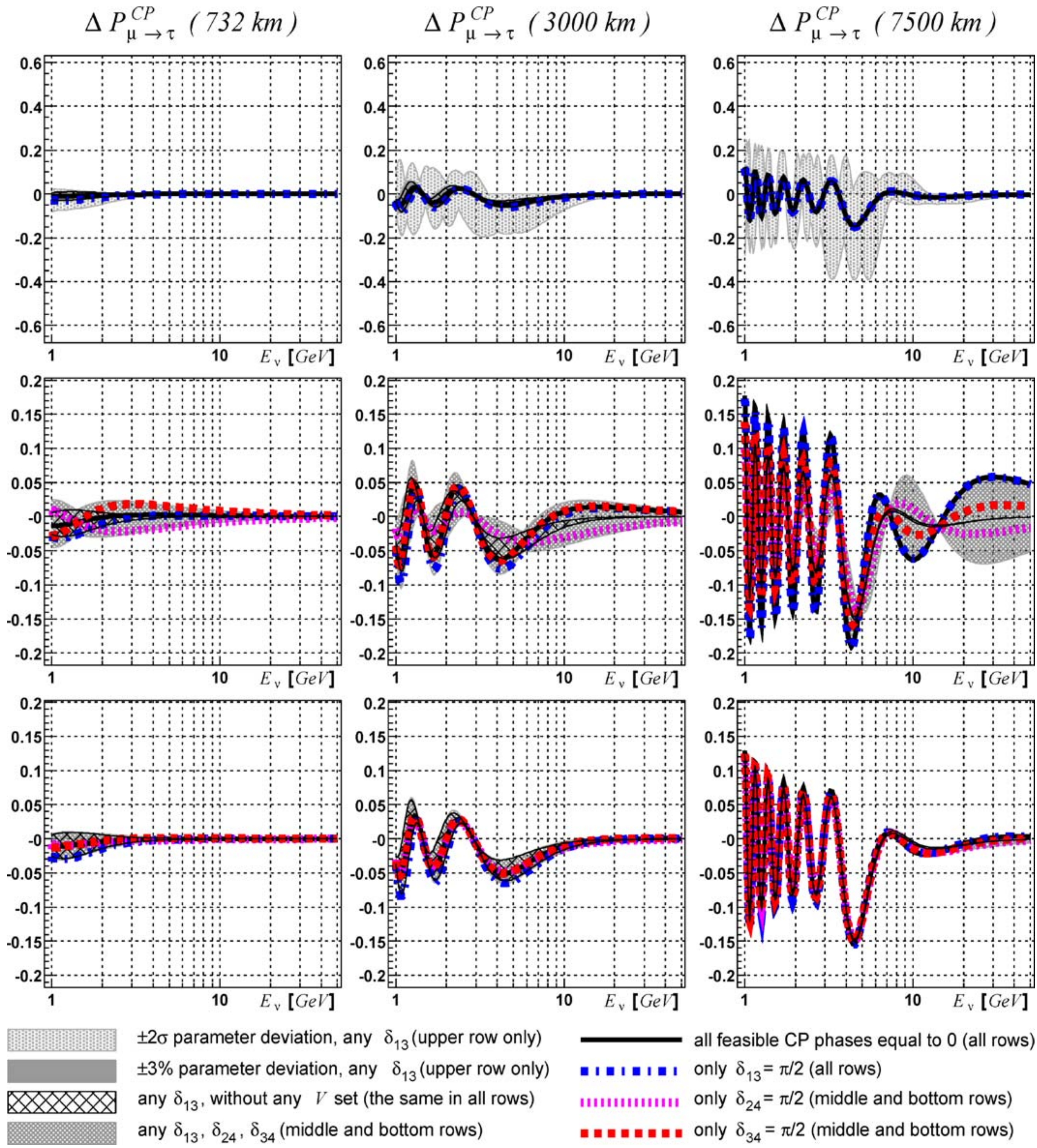

Fig. 1. The probability differences $\Delta P^{CP}_{\mu\to\tau}$ for three baselines $L = 732$, 3000, 7500 km (each column of graphs corresponds to a single L), for the energy range $E_\nu = 1$–50 GeV. Calculations assuming the νSM only are presented in the *upper row* of graphs, whereas results with NP are shown in the *middle* and *bottom rows*, except for the *hashed band*, identical in all rows, which corresponds to the current global best fit parameters with any feasible δ_{13} value and νSM only (no NP). The *dark* (*light*) *gray band* in the *upper row* corresponds to $\pm 3\%$ ($\pm 2\sigma$) deviations of the νSM neutrino oscillation parameters with any feasible δ_{13} value. The *light gray band* in the *middle* (*bottom*) *row* corresponds to the current global best fit parameters with NP V parameters set (40.A) ((40.B)), with any feasible values of $\delta_{13}, \delta_{24}, \delta_{34}$. *Curves* present in all graphs correspond to the current global best fit parameters with all feasible CP phases equal to 0 (*solid curves*) and with exactly one of them equal to $\pi/2$ (*dotted* and *dashed curves*)

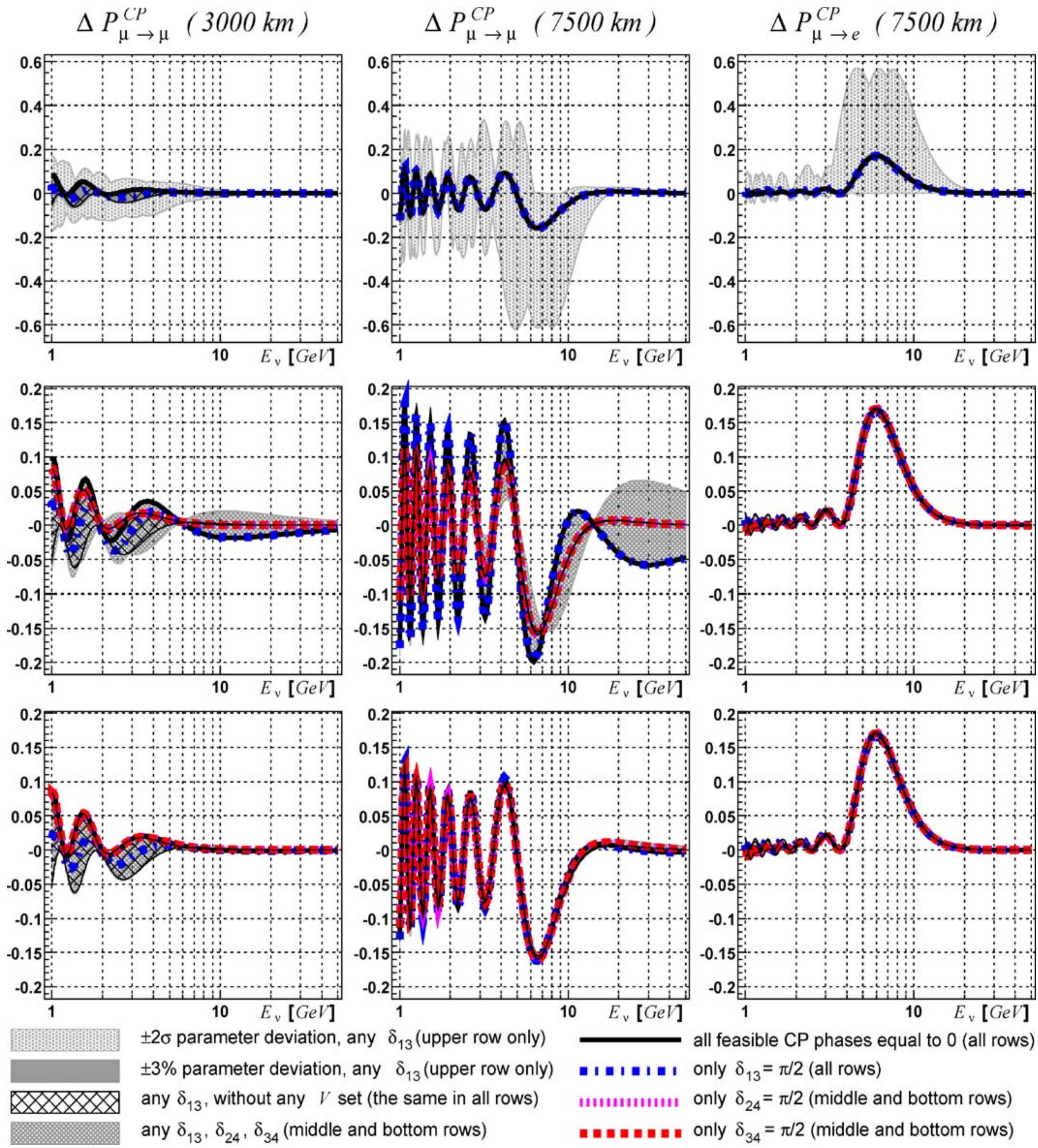

Fig. 2. The probability differences $\Delta P^{CP}_{\mu\to\mu}$ for two baselines $L = 3000$, 7500 km (the *left* and *middle columns* of graphs), and $\Delta P^{CP}_{\mu\to e}$ for $L = 7500$ km (the *right column* of graphs) for the energy range $E_\nu = 1$–50 GeV. Calculations assuming the νSM only are presented in the *upper row* of graphs, whereas results with NP are shown in the *middle* and *bottom rows*, except for the *hashed band*, identical in all rows, which corresponds to the current global best fit parameters with any feasible δ_{13} value and νSM only (no NP). The *dark* (*light*) *gray band* in the *upper row* corresponds to $\pm 3\%$ ($\pm 2\sigma$) deviations of the νSM neutrino oscillation parameters with any feasible δ_{13} value. The *light gray band* in the *middle* (*bottom*) *row* corresponds to the current global best fit parameters with NP V parameters set (40.A) ((40.B)), with any feasible values of $\delta_{13}, \delta_{24}, \delta_{34}$. *Curves* present in all graphs correspond to the current global best fit parameters with all feasible CP phases equal to 0 (*solid curves*) and with exactly one of them equal to $\pi/2$ (*dotted* and *dashed curves*)

NP effects, we have also performed calculations allowing all νSM oscillation parameters to vary by $\pm 3\%$ and $\pm 2\sigma$ (note here that the $\pm 2\sigma$ test is also useful in a qualitative estimation of the effect of the uncertainties in the earth density profile on the νSM results). We have found that, in general, in order to give a chance for the discovery of NP effects, it is required that νSM neutrino oscillation parameter errors should be diminished to the values expected in the future (of about $\pm 3\%$). However, using neutrinos with energies $E_\nu \gtrsim 15\,\mathrm{GeV}$, together with the "magic" baselines mentioned above, it should even be possible to see NP effects with today's νSM neutrino oscillation parameter uncertainties (of about $\pm 2\sigma$). But this chance depends on the actual magnitudes of the NP parameters and the actual precision of the experiment (which may, of course, be required to be much better than the allowed νSM neutrino oscillation parameter errors).

In Fig. 1, the probability differences $\Delta P^{CP}_{\mu\to\tau}$, for the second energy range, and the corresponding baselines set, are shown. The NP effects at $L = 732\,\mathrm{km}$, when scaled by a factor L/E_ν, can be used as an approximate estimation of the expected results for the first energy range with its considered baselines (for which we do not show pictures in this paper).

In the left and middle columns of Fig. 2, the probability differences $\Delta P^{CP}_{\mu\to\mu}$ for the second energy range and the two "magic" Neutrino Factory baselines are shown. For this oscillation channel, the NP effect at $L = 732\,\mathrm{km}$ is significantly smaller than in the $\nu_\mu \to \nu_\tau$ channel at the same distance. This is also the case for this oscillation channel in the first energy range with its considered baselines. Hence, we do not show the corresponding pictures in this paper either.

Note that, in general, the transition probabilities of both neutrino and antineutrino oscillations depend on matter properties in a different way. However, at higher energies the νSM dependence is very much similar; thus any possible uncertainties in the earth density profile cancel in the νSM probability differences (see the upper row of graphs, in both figures, for $E_\nu \gtrsim 15\,\mathrm{GeV}$). In this way, any signal that the probability differences in this energy range are distinctly different from zero will indicate that some NP exists. The statistical significance of such a signal depends on its actual magnitude. For example, for the first V set (40.A), and $L = 7500\,\mathrm{km}$, there exists a maximum at $E_\nu \approx 30\,\mathrm{GeV}$, in which $\Delta P^{CP}_{\mu\to\tau} \approx -\Delta P^{CP}_{\mu\to\mu} \approx \pm 0.06$, while $P_{\mu\to\tau} \approx P_{\mu\to\mu} \approx 0.5$, and thus $\Delta P^{CP}/P \approx \pm 0.12$ (see the middle row of graphs in both figures). The neutrino energy dependent magnitude of this effect can well be reproduced just by the largest term in the $\delta P^{\mathrm{mass}}_{\beta\to\gamma}$ NP correction (responsible for the effective neutrino mass change, see (29)), which is not suppressed either by α, nor by $\sin(2\theta_{13})$. Taking into account the fact that $P_{\overline{\alpha}\to\overline{\beta}}(\delta_{ij}, \widehat{A}_e) = P_{\alpha\to\beta}(-\delta_{ij}, -\widehat{A}_e)$, one can write (see (31))

$$\begin{aligned}\Delta P^{CP}_{\mu\to\tau} &\approx -\Delta P^{CP}_{\mu\to\mu} \approx 2C^0 \\ &\approx 4\widehat{A}_e \sin^3(2\theta_{23}) \Delta \sin(2\Delta) \cos(\chi_{\mu\tau}) \varepsilon_{\mu\tau}\,. \end{aligned} \quad (41)$$

It should be noted, however, that the above formula does not reproduce the $\Delta P^{CP}_{\mu\to\tau}$ well, in case one of the NP feasible CP phases ($\chi_{\mu\tau} = \delta_{34} - \delta_{24}$) is equal to $\pi/2$ (see the dotted and dashed curves in Fig. 1). The reason is that in the above estimation we completely neglect several explicit CP asymmetry enhancing terms, for example ones proportional to $\sin(\chi_{\alpha\beta})$, which are relevant in the $\Delta P^{CP}_{\mu\to\tau}$ case, but which never appear in the case of $\Delta P^{CP}_{\mu\to\mu}$ (see the appendix; note also that these asymmetry terms become dominant at short baselines, and that is why the NP effect at $L = 732\,\mathrm{km}$ is significantly smaller in the $\nu_\mu \to \nu_\mu$ channel than in the $\nu_\mu \to \nu_\tau$ channel at the same distance). From the above formula we can easily learn that the magnitude of this NP effect is linearly proportional to both the actual matter density, through the term $\widehat{A}_e$, and to the $\varepsilon_{\mu\tau}$ NP parameter (thus, as the value of the $\varepsilon_{\mu\tau}$ parameter that results from (40.B) is ten times smaller than the one from (40.A), the NP effect for the second V set (40.B) is smaller by a similar factor, too).

As already mentioned, one of the easiest channels, from the experimental point of view, is the $\nu_\mu \to \nu_e$ one, but it will be difficult to observe any NP there (as it will be in all channels in which ν_e or $\nu_{\bar{e}}$ are involved). In the right column of Fig. 2, the probability differences $\Delta P^{CP}_{\mu\to e}$ for the second energy range and the longest considered baseline are shown (where the biggest effects are expected). It can be seen that the NP effect is rather miserable in this oscillation channel, regardless of the V set given by (40). This conclusion holds also for all other similar oscillation channels and both energy ranges with the corresponding baselines. Moreover, these oscillation channels are sensitive to the (not so very well known) value of $\sin^2(\theta_{13})$. In the other channels, without ν_e and $\nu_{\bar{e}}$, the dependence on $\sin^2(\theta_{13})$ is small, giving a better chance to see the NP effects.

4 Conclusions

In the paper, by introducing a mixing of the low mass "active" neutrinos with heavy ones, we have investigated a possible new physics (NP) scenario that is already present at the TeV scale, that means, at energies close to our present-day experimental facilities. In the presented model (as also in any model with lepton flavor violation), the effective mixing matrix is non-unitary, resulting in non-orthogonal neutrino production and detection states. This leads to the modification of the neutrino oscillations in vacuum. Additionally, non-standard neutrino interactions with matter particles influence the oscillation effects also. First order approximation formulas for the flavor transition probabilities, in constant density matter, for all experimentally available channels, have been given. The possibilities of the experimental verification of the predictions of such a model have been discussed with the prospect of the existing, planned, and feasible Beta Beam, Super Beam, and Neutrino Factory experiments. Numerical calculations of the flavor transition probabilities for two sets (satisfying the present experimental constraints) of the NP parameters that describe a single "effective" heavy neutrino state have been performed. They took into account two energy ranges and several baselines, assuming both the current ($\pm 2\sigma$) and the errors ($\pm 3\%$) expected in the future of today's νSM neutrino oscillation parameters, keeping unchanged their present central values. The realistic PREM I earth density profile model has been applied. One of the easiest channels, from the experimental point of view, is the $\nu_\mu \to \nu_e$ one, but it will be difficult to observe any NP there (as it will also be in all channels in which ν_e or $\nu_{\bar{e}}$ are involved). It appears that the greatest potential for the discovery of any possible presence of NP is in oscillation channels in which ν_e and $\nu_{\bar{e}}$ are not involved at all; that is, in $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_\mu$ (and in the corresponding antineutrino channels). The effects are especially visible for the two so called "magic" Neutrino Factory baselines, $L = 3000$ km and $L = 7500$ km. We have also found that in general, in order to give a chance for the discovery of the NP effects, it is required that the νSM neutrino oscillation parameter errors should be diminished to the values expected in the future (of about $\pm 3\%$). However, using neutrinos with energies $E_\nu \gtrsim 15$ GeV together with the "magic" baselines mentioned above, it should even be possible with today's νSM neutrino oscillation parameter uncertainties (of about $\pm 2\sigma$). But this chance depends on the actual magnitudes of the NP parameters and the actual precision of the experiment (which may, of course, be required to be much better than the allowed νSM neutrino oscillation parameter errors).

Finally, it should be stressed that the full quantitative treatment of the NP effects in future facilities should be based on realistic observables, related, for example, to the expected numbers of events at the corresponding facilities. However, as stated in the beginning of Sect. 2, this would require not only the knowledge of the NP generated transition probability modifications, but also the NP generated modifications of the production and detection neutrino cross sections need to be known. Some preliminary studies [29] suggest that the expected effects on the last two terms can be of the same order as these on the first term (shown in this paper).

Acknowledgements. This work has been supported by the Polish Ministry of Science under Grant 1P03B04926, and by the European Community's Marie-Curie Research Training Network under contract MRTN-CT-2006-035505 "Tools and Precision Calculations for Physics Discoveries at Colliders".

Appendix: First order approximations for flavor transition probabilities

Here we collect all formulas for the flavor transition probabilities, in constant density matter, for all experimentally available neutrino and antineutrino channels. As already mentioned, in all formulas, we assume $\delta m^2_{21} = \delta m^2_{\mathrm{sol}}$, and $\delta m^2_{31} = \pm\delta m^2_{\mathrm{atm}} + \delta m^2_{\mathrm{sol}}/2$, where the upper (lower) sign refers to the normal (inverted) mass hierarchy [98, 99].

Firstly, we show formulas required in order to calculate the c term in (22), which is responsible for the initial and final neutrino state modifications [95]:

$$\begin{aligned}\delta P^c_{\alpha\to\beta}(L) = (c_{\alpha\alpha} + c_{\beta\beta}) P^{\mathrm{SM}}_{\alpha\to\beta}(L) - 2\,\mathrm{Re}(c_{\alpha\beta})\\ - 4\sum_{i>k} \mathrm{Re}\left[(\delta\widetilde{T}^c)^{ik}_{\alpha\beta}\right] \sin^2\left(\frac{\widetilde{\Delta}_{ik}}{2}\right)\\ - 2\sum_{i>k} \mathrm{Im}\left[(\delta\widetilde{T}^c)^{ik}_{\alpha\beta}\right] \sin\left(\widetilde{\Delta}_{ik}\right),\end{aligned} \quad (A.1)$$

where

$$\begin{aligned}(\delta\widetilde{T}^c)^{ik}_{\alpha\beta} = -\sum_\gamma \left(\delta\lambda_{\alpha\gamma}\widetilde{U}_{\gamma i}\widetilde{U}_{\beta k} + \delta\lambda_{\beta\gamma}\widetilde{U}_{\alpha i}\widetilde{U}_{\gamma k}\right)\widetilde{U}^*_{\alpha k}\widetilde{U}^*_{\beta i}\\ - \sum_\gamma \left(\delta\lambda^*_{\alpha\gamma}\widetilde{U}^*_{\gamma k}\widetilde{U}^*_{\beta i} + \delta\lambda^*_{\beta\gamma}\widetilde{U}^*_{\alpha k}\widetilde{U}^*_{\gamma i}\right)\widetilde{U}_{\alpha i}\widetilde{U}_{\beta k},\end{aligned} \quad (A.2)$$

and

$$\widetilde{\Delta}_{ik} = \frac{\delta\widetilde{m}^2_{ij} L}{2E_\nu}. \quad (A.3)$$

Here, $\widetilde{U}_{\alpha i}$ and $\delta\widetilde{m}^2_{ij}$ are, respectively, the effective mixing matrix elements and the effective mass square differences, coming from the diagonalization of the νSM Hamiltonian in matter (without any NP; see, for example, [110]), and $\delta\lambda_{\alpha\beta}$ can be calculated by putting $\delta\Lambda = \mathbf{1} - \mathcal{U}U^\dagger$ (compare (10)).

Secondly, as described in Sect. 2, not all available channels were discussed in this paper. We were only interested in the $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_\mu$ channels, including the corresponding antineutrino transitions. In this appendix, however, we collect all required equations.

In any case, the total transition probability is decomposed into several terms according to (21)–(23), (28) and (29). In general, the mass term $\delta P^{\mathrm{mass}}_{\beta\to\gamma}$ ((23) and (29)) does not vanish only for channels in which ν_e and $\nu_{\bar{e}}$ do not enter. This term is also, up to the sign, the same for all such channels. Let us put

$$\begin{aligned}C^\alpha = \Delta\sin(2\Delta)\sin(2\theta_{12})\sin^2(2\theta_{23})\\ \times\left[\cos(\theta_{23})\cos(\chi_{e\mu})\varepsilon_{e\mu} - \cos(\chi_{e\tau})\sin(\theta_{23})\varepsilon_{e\tau}\right],\end{aligned} \quad (A.4)$$

$$\begin{aligned}C^s = \frac{\widehat{A}_e}{1-\widehat{A}_e}\left\{\Delta\sin(2\Delta)\sin^2(2\theta_{23})\right.\\ \times\left[\cos(\delta_{13}+\chi_{e\mu})\sin(\theta_{23})\varepsilon_{e\mu}\right.\\ \left.\left.+\cos(\theta_{23})\cos(\delta_{13}+\chi_{e\tau})\varepsilon_{e\tau}\right]\right\}.\end{aligned} \quad (A.5)$$

For the $\nu_\mu \to \nu_e$ transition, we obtain

$$\delta P^{\mathrm{mass}}_{\mu\to e} = 0, \quad B^0_{\mu e} = 0, \quad (A.6)$$

and two non-vanishing terms have the following form:

$$\begin{aligned}B_{\mu e}^{\alpha} &= \frac{1}{(\widehat{A}_e-1)\widehat{A}_e^2}\cos(\theta_{23})\sin(2\theta_{12}) \\ &\times \{\varepsilon_{e\tau}\cos(\theta_{23})\sin(\theta_{23})[\cos(\chi_{e\tau})[\widehat{A}_e\cos(2\Delta) \\ &-\widehat{A}_e\cos(2(\widehat{A}_e-1)\Delta)-2(\widehat{A}_e-2)\sin^2(\widehat{A}_e\Delta)] \\ &+\sin(\chi_{e\tau})\widehat{A}_e \\ &\times[\sin(2\Delta)-\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\widehat{A}_e\Delta)]] \\ &+\varepsilon_{e\mu}[2\cos(\chi_{e\mu})\sin(\widehat{A}_e\Delta) \\ &\times[2(\widehat{A}_e-1)\cos^2(\theta_{23})\sin(\widehat{A}_e\Delta) \\ &+\widehat{A}_e(\sin((\widehat{A}_e-2)\Delta)+\sin(\widehat{A}_e\Delta))\sin^2(\theta_{23})] \\ &+\sin(\chi_{e\mu})\widehat{A}_e\sin^2(\theta_{23}) \\ &\times[\sin(2\Delta)-\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\widehat{A}_e\Delta)]]\}, \end{aligned} \tag{A.7}$$

$$\begin{aligned}B_{\mu e}^{s} &= \frac{1}{(\widehat{A}_e-1)\widehat{A}_e^2}\sin(\theta_{23})\{\varepsilon_{e\tau}\sin(\theta_{23})\cos(\theta_{23}) \\ &\times[\cos(\delta_{13}+\chi_{e\tau})[1+\widehat{A}_e-(\widehat{A}_e-1)\cos(2\Delta) \\ &-\cos(2(\widehat{A}_e-1)\Delta)-(1-\widehat{A}_e)\cos(2\widehat{A}_e\Delta)] \\ &+\sin(\delta_{13}+\chi_{e\tau})(\widehat{A}_e-1) \\ &\times[\sin(2\Delta)-\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\widehat{A}_e\Delta)]] \\ &+\varepsilon_{e\mu}[\cos(\delta_{13}+\chi_{e\mu}) \\ &\times[(\widehat{A}_e-1)\cos^2(\theta_{23})(\cos(2\Delta)-\cos(2(\widehat{A}_e-1)\Delta) \\ &+2\sin^2(\widehat{A}_e\Delta))+4\widehat{A}_e\sin^2((\widehat{A}_e-1)\Delta)\sin^2(\theta_{23})] \\ &+\sin(\delta_{13}+\chi_{e\mu})(\widehat{A}_e-1)\cos^2(\theta_{23}) \\ &\times[\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\Delta)+\sin(2\widehat{A}_e\Delta)]]\}. \end{aligned} \tag{A.8}$$

For the $\nu_\mu \to \nu_\tau$ transition, all terms are different from zero. According to the previous discussion, the leading term $B_{\mu\tau}^0 = B^0$ (see (33)) and the two non-leading terms are given by

$$\begin{aligned}B_{\mu\tau}^{\alpha} &= \frac{\sin(2\theta_{23})}{2(-1+\widehat{A}_e)\widehat{A}_e}\{\varepsilon_{e\mu}\sin(2\theta_{12})\sin(\theta_{23}) \\ &\times[\cos(\chi_{e\mu})[[\cos(2(-1+\widehat{A}_e)\Delta)-\cos(2\widehat{A}_e\Delta)] \\ &\times(1-\widehat{A}_e+\cos(2\theta_{23})) \\ &+2(2-\widehat{A}_e-2\widehat{A}_e^2)\cos^2(\theta_{23})\sin^2(\Delta) \\ &+(4\widehat{A}_e^2-2\widehat{A}_e)\sin^2(\theta_{23})\sin^2(\Delta)] \\ &-\widehat{A}_e\sin(\chi_{e\mu}) \\ &\times[\sin(2\Delta)+\sin(2(1-\widehat{A}_e)\Delta)+\sin(2\widehat{A}_e\Delta)]] \\ &+\varepsilon_{e\tau}\cos(\theta_{23})\sin(2\theta_{12})[\cos(\chi_{e\tau}) \\ &\times[[\cos(2(-1+\widehat{A}_e)\Delta)-\cos(2\widehat{A}_e\Delta)] \\ &\times(-1+\widehat{A}_e+\cos(2\theta_{23})) \\ &+2\widehat{A}_e\cos^2(\theta_{23})\sin^2(\Delta)-4\widehat{A}_e^2\cos(2\theta_{23})\sin^2(\Delta) \\ &+2(\widehat{A}_e-2)\sin^2(\Delta)\sin^2(\theta_{23})] \\ &+\widehat{A}_e\sin(\chi_{e\tau}) \\ &\times[-\sin(2\Delta)+\sin(2(1-\widehat{A}_e)\Delta)+\sin(2\widehat{A}_e\Delta)]] \\ &-4(-1+\widehat{A}_e)\widehat{A}_e^2\cos^2(\theta_{12})\cos(2\theta_{23}) \\ &\times(2\Delta\cos(\Delta)-\sin(\Delta))\sin(\Delta) \\ &\times[\sin(2\theta_{23})(\varepsilon_{\mu\mu}-\varepsilon_{\tau\tau})+2\cos(2\theta_{23})\cos(\chi_{\mu\tau})\varepsilon_{\mu\tau}]\}, \end{aligned} \tag{A.9}$$

$$\begin{aligned}B_{\mu\tau}^{s} &= \frac{\sin(2\theta_{23})}{2(-1+\widehat{A}_e)^2}\{\varepsilon_{e\mu}\cos(\theta_{23})[2\cos(\delta_{13}+\chi_{e\mu})\sin(\Delta) \\ &\times[(-\widehat{A}_e-(1-4\widehat{A}_e+2\widehat{A}_e^2)\cos(2\theta_{23}))\sin(\Delta) \\ &-(\widehat{A}_e-\cos(2\theta_{23}))\sin(\Delta-2\widehat{A}_e\Delta)] \\ &+(-1+\widehat{A}_e)\sin(\delta_{13}+\chi_{e\mu}) \\ &\times[\sin(2\Delta)-\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\widehat{A}_e\Delta)]] \\ &+\varepsilon_{e\tau}\sin(\theta_{23})[2\cos(\delta_{13}+\chi_{e\tau})\sin(\Delta) \\ &\times[(-\widehat{A}_e+(1-4\widehat{A}_e+2\widehat{A}_e^2)\cos(2\theta_{23}))\sin(\Delta) \\ &-(\widehat{A}_e+\cos(2\theta_{23}))\sin(\Delta-2\widehat{A}_e\Delta)] \\ &-(-1+\widehat{A}_e)\sin(\delta_{13}+\chi_{e\tau}) \\ &\times[\sin(2\Delta)-\sin(2(1-\widehat{A}_e)\Delta)-\sin(2\widehat{A}_e\Delta)]]\}. \end{aligned} \tag{A.10}$$

For the mass correction terms, the first one, $C_{\mu\tau}^0$, is given by (33), and the two other universal terms are the following:

$$C_{\mu\tau}^{\alpha} = C^{\alpha}, \quad C_{\mu\tau}^{s} = C^{s}. \tag{A.11}$$

Also for the $\nu_\mu \to \nu_\mu$ transition, all terms contribute. The modulus of the first interaction correction term is given by (33), whereas the two remaining terms are as follows:

$$\begin{aligned}B_{\mu\mu}^{\alpha} &= \frac{2}{(-1+\widehat{A}_e)\widehat{A}_e}\{\varepsilon_{e\mu}\cos(\chi_{e\mu})\cos(\theta_{23})\sin(2\theta_{12}) \\ &\times[(-1+\widehat{A}_e)\cos(2\widehat{A}_e\Delta)\cos^4(\theta_{23}) \\ &+\widehat{A}_e\sin^2(\theta_{23})(-1+\cos(2(-1+\widehat{A}_e)\Delta)\sin^2(\theta_{23}) \\ &-2(-1+\widehat{A}_e)\sin^2(\Delta)\sin^2(\theta_{23})) \\ &+\cos^2(\theta_{23})(1-\widehat{A}_e \\ &+(-1+\widehat{A}_e)\cos(2(-1+\widehat{A}_e)\Delta)\sin^2(\theta_{23}) \\ &+\widehat{A}_e\cos(2\widehat{A}_e\Delta)\sin^2(\theta_{23}) \\ &-2\sin^2(\Delta)\sin^2(\theta_{23})+2\widehat{A}_e\sin^2(\Delta)\sin^2(\theta_{23}) \\ &+2\widehat{A}_e^2\sin^2(\Delta)\sin^2(\theta_{23}))] \\ &+\frac{1}{2}\varepsilon_{e\tau}\cos(\chi_{e\tau})\cos(\theta_{23})\sin(2\theta_{12}) \\ &\times[2\cos(2\widehat{A}_e\Delta)\cos^3(\theta_{23})\sin(\theta_{23}) \\ &+2\cos(2(-1+\widehat{A}_e)\Delta)\cos(\theta_{23})\sin^3(\theta_{23}) \\ &+4\cos(\theta_{23})\sin^2(\Delta)\sin^3(\theta_{23})-\sin(2\theta_{23}) \\ &+\widehat{A}_e^2\sin^2(\Delta)\sin(4\theta_{23})] \\ &+(-1+\widehat{A}_e)\widehat{A}_e^2\cos^2(\theta_{12})(2\Delta\cos(\Delta)-\sin(\Delta)) \\ &\times\sin(\Delta)\sin(4\theta_{23}) \\ &\times[\sin(2\theta_{23})(\varepsilon_{\mu\mu}-\varepsilon_{\tau\tau})+2\cos(2\theta_{23})\cos(\chi_{\mu\tau})\varepsilon_{\mu\tau}]\}, \end{aligned} \tag{A.12}$$

$$
\begin{aligned}
B^s_{\mu\mu} = &\frac{2}{\left(-1+\widehat{A}_e\right)^2}\left\{\varepsilon_{e\mu}\cos(\delta_{13}+\chi_{e\mu})\right.\\
&\times\left[\frac{\sin(\theta_{23})}{2}\left(-1+2\widehat{A}_e-\cos(2\theta_{23})\right)\right.\\
&\times\left(-1+\cos\left(2\widehat{A}_e\Delta\right)\cos^2(\theta_{23})\right.\\
&\left.+\cos\left(2\left(-1+\widehat{A}_e\right)\Delta\right)\sin^2(\theta_{23})\right)\\
&+\cos(\theta_{23})\sin^2(\Delta)\left(\left(-1+\widehat{A}_e\right)\widehat{A}_e\cos^2(\theta_{23})\right.\\
&\left.\left.-\left(1+\left(-3+\widehat{A}_e\right)\widehat{A}_e\right)\sin^2(\theta_{23})\right)\sin(2\theta_{23})\right]\\
&+\varepsilon_{e\tau}\cos(\delta_{13}+\chi_{e\tau})\sin(\theta_{23})\\
&\times\left[-2\cos^3(\theta_{23})\sin^2\left(\widehat{A}_e\Delta\right)\sin(\theta_{23})\right.\\
&-2\cos(\theta_{23})\sin^2\left(\Delta-\widehat{A}_e\Delta\right)\sin^3(\theta_{23})\\
&+\sin^2(\Delta)\\
&\left.\left.\times\left(\left(2-\widehat{A}_e\right)\widehat{A}_e\cos(2\theta_{23})+\sin^2(\theta_{23})\right)\sin(2\theta_{23})\right]\right\}. \qquad \text{(A.13)}
\end{aligned}
$$

The mass term for this channel has the opposite sign, in comparison to the previously discussed transition $\nu_\mu \to \nu_\tau$:

$$\delta P^{\text{mass}}_{\mu\to\mu} = -\delta P^{\text{mass}}_{\mu\to\tau}\,. \qquad \text{(A.14)}$$

Finally, the νSM probabilities are as follows:

$$
\begin{aligned}
P^{\text{SM}}_{\mu\to e} = &\frac{\alpha^2}{\widehat{A}_e^2}\left(\cos^2(\theta_{23})\sin^2\left(\widehat{A}_e\Delta\right)\sin^2(2\theta_{12})\right)\\
&+\frac{\sin^2(2\theta_{13})}{\left(-1+\widehat{A}_e\right)^2}\left(\sin^2\left(\Delta-\widehat{A}_e\Delta\right)\sin^2(\theta_{23})\right)\\
&-\frac{\alpha\sin(2\theta_{13})}{\left(-1+\widehat{A}_e\right)\widehat{A}_e}\\
&\times\left\{\left[\cos(\delta_{13})\cos(\Delta)+\sin(\delta_{13})\sin(\Delta)\right]\right.\\
&\left.\times\sin\left(\widehat{A}_e\Delta\right)\sin\left(\Delta-\widehat{A}_e\Delta\right)\sin(2\theta_{12})\sin(2\theta_{23})\right\}, \qquad \text{(A.15)}
\end{aligned}
$$

$$
\begin{aligned}
P^{\text{SM}}_{\mu\to\tau} = &\sin^2(\Delta)\sin^2(2\theta_{23})\\
&-\alpha\Delta\cos^2(\theta_{12})\sin(2\Delta)\sin^2(2\theta_{23})\\
&+\frac{\alpha^2}{4\widehat{A}_e^2}\\
&\times\left\{\sin(\Delta)\sin\left(\Delta-2\widehat{A}_e\Delta\right)\sin^2(2\theta_{12})\sin^2(2\theta_{23})\right\}\\
&-\frac{\sin^2(2\theta_{13})}{4\left(-1+\widehat{A}_e\right)^2}\left\{\sin(\Delta)\left[2\left(\widehat{A}_e-1\right)\widehat{A}_e\Delta\cos(\Delta)\right.\right.\\
&\left.\left.+\sin(\Delta)+\sin\left(\Delta-2\widehat{A}_e\Delta\right)\right]\sin^2(2\theta_{23})\right\}\\
&+\frac{\alpha\sin(2\theta_{13})}{2\left(-1+\widehat{A}_e\right)\widehat{A}_e}\left\{-\cos(\delta_{13})\left[\cos(2\theta_{23})\sin(\Delta)\right.\right.\\
&\times\left(\left(-1+2\widehat{A}_e^2\right)\sin(\Delta)+\sin\left(\Delta-2\widehat{A}_e\Delta\right)\right)\\
&\left.\times\sin(2\theta_{12})\sin(2\theta_{23})\right]\\
&-2\sin(\delta_{13})\left[\sin(\Delta)\sin\left(\widehat{A}_e\Delta\right)\sin\left(\Delta-\widehat{A}_e\Delta\right)\right.\\
&\left.\left.\times\sin(2\theta_{12})\sin(2\theta_{23})\right]\right\}, \qquad \text{(A.16)}
\end{aligned}
$$

$$
\begin{aligned}
P^{\text{SM}}_{\mu\to\mu} = &1-\sin^2(\Delta)\sin^2(2\theta_{23})\\
&+\alpha\Delta\cos^2(\theta_{12})\sin(2\Delta)\sin^2(2\theta_{23})\\
&+\frac{\sin^2(2\theta_{13})}{4\left(-1+\widehat{A}_e\right)^2}\left\{-4\sin^2\left(\Delta-\widehat{A}_e\Delta\right)\sin^4(\theta_{23})\right.\\
&+\left(\sin^2(\Delta)+\left(-1+\widehat{A}_e\right)\widehat{A}_e\Delta\sin(2\Delta)\right.\\
&\left.\left.-\sin^2\left(\widehat{A}_e\Delta\right)\right)\sin^2(2\theta_{23})\right\}\\
&-\frac{\alpha^2}{4\widehat{A}_e^2}\sin^2(2\theta_{12})\left\{4\cos^4(\theta_{23})\sin^2\left(\widehat{A}_e\Delta\right)\right.\\
&\left.+\sin^2\left(\Delta-\widehat{A}_e\Delta\right)\sin^2(2\theta_{23})\right\}\\
&+\frac{\alpha\sin(2\theta_{13})}{\left(-1+\widehat{A}_e\right)\widehat{A}_e}\cos(\delta_{13})\sin(2\theta_{12})\\
&\times\left\{\sin^2(\Delta)\left(\widehat{A}_e^2\cos(2\theta_{23})+\sin^2(\theta_{23})\right)\sin(2\theta_{23})\right.\\
&-\sin(\theta_{23})\left(2\cos^3(\theta_{23})\sin^2\left(\widehat{A}_e\Delta\right)\right.\\
&\left.\left.+\sin^2\left(\Delta-\widehat{A}_e\Delta\right)\sin(\theta_{23})\sin(2\theta_{23})\right)\right\}. \qquad \text{(A.17)}
\end{aligned}
$$

The formulas for the time reversed channels, $\nu_e \to \nu_\mu$ and $\nu_\tau \to \nu_\mu$, are obtained from the above formulas after the replacements $\Delta \to -\Delta$ and $\widetilde{\Delta}_{ik} \to -\widetilde{\Delta}_{ik}$. Finally, if neither scalar nor pseudoscalar neutrino interactions are considered (like in the model that we present here [29]), for antineutrinos the interaction Hamiltonian H is replaced by $-H^*$; thus the transitions formulas for the channels $\nu_{\bar\mu} \to \nu_{\bar e}$, $\nu_{\bar\mu} \to \nu_{\bar\tau}$ and $\nu_{\bar\mu} \to \nu_{\bar\mu}$ are easily obtained after the replacements $\delta_{13} \to -\delta_{13}$, $\widehat{A}_e \to -\widehat{A}_e$ and $\delta\lambda_{\alpha\beta} \to \delta\lambda^*_{\alpha\beta}$ (hence also $\chi_{\alpha\beta} \to -\chi_{\alpha\beta}$). This completes the set of νSM transition probability formulas and the NP corrections to them, for all experimentally available channels.

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